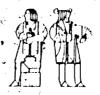
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LABORATORY FOR COMPUTER SCIENCE



MASSACHUSETTS
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## REPORT ON THE FX-91 PROGRAMMING LANGUAGE

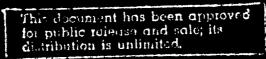


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February 1992



### Report on the FX-91 Programming Language

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#### SUMMARY

This report gives a defining description of the programming language FX-91. The FX (short for FX-91) programming language is designed to support the parallel implementation of applications that perform both symbolic and scientific computations. The unique features of FX include:

- An effect system, to discover expression scheduling constraints. An effect is a static description of the side-effects an expression may perform when it is evaluated. Just as a type describes what an expression computes, an effect describes how an expression computes.
- Abstraction over any kind of description, thus permitting first-class type and effect polymorphism. Effect polymorphism makes the FX effect system more powerful than previous approaches to side-effect analysis in the presence of first-class subroutines.
- Type and effect inference, so that declaration free programs can be statically type and effect checked. FX
  also permits explicitly typed programs, and programs
  that use explicit types only for first-class polymorphic
  values and modules.
- First-class modules, which permit FX to serve as its own configuration language. It also includes an architecture independent module of parallel vector operators.

The introduction offers a summary of and motivation for the unique properties of FX-91.

- Chapter 1 presents the fundamental ideas of the language and describes the notational conventions used for describing the language and for writing programs in the language.
- Chapter 2 describes the FX-91 Kernel. The FX Kernel includes essential constructs and the type and effect system.
- Chapter 3 introduces built-in data types and operations, which include all of the language's data manipulation and input-output primitives.

#### CONTENTS

Introduction	2
1. Overview of $FX$	3
1.1. Semantics	3
1.2. Lexicon	3
1.3. Static and Dynamic Errors	4
1.4. Conventions	4
2. The FX-91 Kernel	5
2.1. Kinds	5
2.2. Descriptions	5
2.3. Values	8
	13
3. Standard Descriptions	16
	16
	16
	16
3.4. Write	16
3.5. Unit	16
3.6. Bool	16
	16
3.8. Float	17
3.9. Char	17
	18
	18
	19
	19
	19
	19
	<b>2</b> 0
	- ı 21
	$\frac{-1}{22}$
	23
Alphabetic index of definitions of concepts,	
	24

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Pierre Jouvelot is also with the Centre de Recherche en Informatique of Ecole des Mines de Paris, France.

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Programming Systems Research Group Laboratory for Computer Science

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Keywords: Programming Languages, Types, Effects, Inference, Polymorphism, Modules, Static Checking, Vector Operations

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#### INTRODUCTION

FX-91 is a programming language that we designed to investigate the following questions:

- How can simple syntactic rules be used to deduce program properties beyond type information?
- How important is information about the side-effects of program expressions in a language that is designed for parallel computing, and to what extent can unambiguous side-effect information be used to schedule a program for parallel execution?
- How important are first-class polymorphic values and first-class modules in a language that provides type inference?

FX-91 is a major revision and extension of the FX-87 programming language [GJLS87]. The designs of both FX-91 and FX-87 were strongly influenced by Scheme [R86], especially in the choice of standard types and operations.

FX-87 was the first programming language to incorporate an effect system [LG88]. Experimental data from FX-87 programs show that effect information can be used to automatically schedule imperative programs for parallel execution [HG88]. However, we found that FX-87 was difficult to use because extensive declarations were required in programs.

FX-91 is designed to be easier to use than FX-87. FX-91 eliminates the requirement for most declarations [OG89, JG91], provides a less complex effect system, and provides a module system that supports programming in the large [SG90].

We have found that an effect system is useful to programmers, compiler writers, and language designers in the following respects:

- An effect system lets the programmer specify the sideeffect properties of program modules in a way that is
  machine-verifiable. The resulting effect specifications
  are a natural extension of the type specifications found
  in conventional programming languages. We believe
  that effect specifications have the potential to improve
  the design and maintenance of imperative programs.
- An effect system lets the compiler identify optimization opportunities that are hard to detect in a conventional higher-order imperative programming language. We have focused our research on three classes of optimizations: execution time (including eager, lazy, and parallel evaluation); common subexpression elimination (including memoization); and dead code elimination. We believe that the ability to perform these optimizations effectively in the presence of side-effects represents a step towards integrating functional and imperative programming for the purpose of parallel programming.

• An effect system lets the language designer express and enforce side-effect constraints in the language definition. In FX, for example, the body of a polymorphic expression must not have any side-effects. This restriction makes FX the first language known to us that permits an efficient implementation of fully orthogonal polymorphism in the presence of side-effects. In FX, any expression can be abstracted over any type and all polymorphic values are first-class. First-class values can be passed to subroutines, returned from subroutines, and placed in the store.

The FX-91 programming language was developed by the Programming Systems Research Group at MIT. In addition to the authors, Jonathan Rees and Franklyn Turbak contributed to the design of FX-91. Any information or comments about FX-91 can be submitted to the FX electronic mailing list fxelcs.mit.edu. Send requests to be added to the list to fx-requestelcs.mit.edu.

An FX-91 interpreter written in Scheme can be obtained by sending an electronic mail request to fx-request@lcs.mit.edu.

#### DESCRIPTION OF THE LANGUAGE

#### 1. Overview of FX

FX uses lambda abstraction and beta-reduction as the basis of its computational model, and thus it is a member of the lambda calculus family of languages. FX uses symbolic expression (s-expression) syntax, and thus it is compatible with Lisp source maintenance tools. FX is lexically scoped, statically checked, uses one variable namespace and implements tail-recursion. All values in FX are first-class, including subroutines, polymorphic values and modules.

The FX programming system is based on a kernel language that defines the syntax and semantics of a core set of primitive FX expressions. The kernel is primitive in the sense that it defines twenty different value expressions, and there is no simple way to express these expressions in terms of one another. Thus the FX kernel forms the core of the FX programming system from the point of view of both the FX application programmer and the FX language implementor.

The foundation provided by the FX kernel is supplemented with a library of standard types and operators that are contained in the fx module. The fx module contains types and operations for booleans, integers, floating point numbers, characters, strings, symbols, permutations, unique values, lists, vectors, symbolic expressions and input-output streams. The fx module can be defined in terms of kernel expressions, and can be replaced by programmers who wish to change the implementation of standard types.

#### 1.1. Semantics

The semantic definition of the FX kernel is divided into a static semantics that is used to deduce the properties of programs before they are run and a dynamic semantics that describes the behavior of programs at execution time.

There are two key theorems that relate the static and dynamic semantics of FX. The type soundness theorem guarantees that the static type of an expression (the type computed by the static semantics) will be a conservative approximation of its dynamic type (the type of the value computed by the dynamic semantics). The effect soundness theorem guarantees that the static effect of an expression will be a conservative approximation of its dynamic effect. These theorems permit results from the static semantics to be used by FX implementations to improve dynamic performance.

The FX static semantics is based on a hierarchical kinded type system that includes kinds, universal polymorphism, higher order types, and recursive types. The static semantics describes expressions with description expressions. There are two principle kinds of descriptions: types, which describe the values expressions compute, and effects, which

describe the side-effects of expressions. An expression may be polymorphic in any kind of description. Thus the type of a subroutine may depend on the effect parameters passed to it. Effect polymorphism permits the static semantics to provide tight effect bounds on higher-order functionals in a natural and simple manner.

The FX static semantics will reconstruct omitted type and effect declarations in a manner that combines the implicit typing of ML[MTH90] with the full power of the explicitly typed second-order polymorphic lambda calculus. The FX reconstruction system relieves the programmer of the burden of providing type and effect declarations while retaining the benefits of strongly-typed languages, including superior performance, documentation, and safety. The FX type reconstruction system will accept ML-style programs, explicitly typed programs, and programs that use explicit types only for first-class polymorphic values and modules. We offer this flexibility by providing both generic and explicitly-quantified polymorphic types in FX, along with an operator to convert between these two forms of polymorphism.

The FX static semantics provides complete checking of module values. The FX module system permits types and values to be packaged as first-class module values. Because modules are first-class values, FX does not require a separate configuration language.

#### 1.2. Lexicon

The basic lexical entities used in the FX programming language are the following:

- A digit is one of 0 ... 9.
- A letter is one of a ... z or A .. Z.
- The set of extended alphabetic characters must include: \*, /, <, =, >, !, ?, :, \$, %, \_, &, -, -, [, ], &.
- A white space is a blank space a newline character, a tab character, or a newpage character.
- A character is a digit, a letter, an extended alphabetic character, +, -, a white space or backspace character.
- A delimiter is a white space, a left parenthesis or a right parenthesis.
- A token is a sequence of characters that is separated by delimiters.
- A number is a token made of a non-empty sequence of digits, possibly including base and exponent information, a decimal point, and a sign. (see Chapter 3).

#### 4 FX Report

- A literal is either a number, or a token that begins
  with ' or #, or a sequence of characters or \ enclosed
  in double quotes ", or the symbol keyword and an
  identifier enclosed in parentheses.
- An identifier is a token beginning with a letter or extended alphabetic character and made of a non-empty sequence of letters, digits, extended alphabetic characters, and the characters + and -. Note that + and by themselves are also identifiers. Identifiers are case-insensitive.

FX reserves the following identifiers. Reserved identifiers must not be bound, redefined, or used as tags for sums.

abs	and	begin
cond	define-abstraction	define
define-datatype	define-description	define-typed
desc	dlambda	effect
else	extend	extract
fx	if	lambda
let	letrec	let*
load	match	maxeff
module	moduleof	open
or	plambda	poly
product	productof	proj
select	sum	sumof
symbol	tagcase	the
type	val	with

Comments in FX are sequences of characters beginning with a ";" and ending with the end of the line on which the ";" is located. They are discarded by FX and treated as a single whitespace.

#### 1.3. Static and Dynamic Errors

Static errors are detected by the FX static semantics. All syntax, type, and effect errors are detected statically and reported. The sentence "x must be y" indicates that "it is a static error if x is not y".

Dynamic errors may be detected by FX when a program is run. The phrase "a dynamic error is signalled" indicates that FX implementations must report the corresponding dynamic error and proceed in an implementation-dependent manner. The phrase "it is a dynamic error" indicates that FX implementations do not have to detect or report the corresponding dynamic error. The meaning of a program that contains a dynamic error is undefined.

#### 1.4. Conventions

This report adheres to the following conventions:

FX program text is written in teletype font. Program text is comprised of identifiers, literals, and delimiters.

- Meta-expressions, which are name: for syntactic classes of expressions, are written in italic font. A programmer may replace any meta-expression by a compatible FX expression.
- Certain FX language forms have a variable number of components. A possibly empty sequence of n expressions is noted  $e_1...e_n$  or  $..._{i=1}^n e_i$ . If the name of the upper bound on subscripts is not used, we write the shorter:  $e_1...$  If there is at least one expression in the sequence (i.e.  $n \ge 1$ ), we use  $e_1e_2...e_n$ . We usually denote by  $e_i$  (or any other subscripted e) an expression belonging to such sequence. Certain parameters can have different forms; [x|y] stands for either x or y.
- The set of values x that satisfy the predicate P is noted  $\{x \mid P(x)\}$ ; predicates are defined as usual. The difference of two sets S and T is noted S T. For an ordered index set S, we note  $\{x \in Se_x\}$  the set of  $e_x$  for each x of S. As a shorthand,  $\{i \in [1,n]e_i\}$  is noted  $\{i_{i=1}^n e_i\}$ . The interval of ordered values between x and y is noted [x,y]. If the lower bound is excluded, [x,y] is used instead; the same convention applies to upper bounds.
- The function that is equal to the function f, except at x (not in the domain of f) where it yields y, is noted  $f[x \mapsto y]$ . As a short hand, we note  $f[_{i=1}^n x_i \mapsto y_i]$  the function equal to f, except at each of the pairwise distinct n arguments  $x_i$  where it yields  $y_i$ . The result of the application of f to x is noted fx.
- A variable is free in an expression e if it does not appear in any of the binding constructs within e. A variable that is not free is bound. (Binding constructs are labelled as such in their definition.) All bound variables are alpha-renamed to avoid name clashes with the surrounding context.
- The syntactic substitution s of the variable id by the expression e (noted  $\lfloor e/id \rfloor$ ) is the function, defined by induction on the syntactic structure of its domain, that substitutes any free appearance of id in its argument by e; alpha-renaming of bound variables is performed to avoid name clashes. For an ordered index set S, we write  $\lfloor x \in Se_x/id_x \rfloor$  for the successive substitutions of  $id_x$  by  $e_x$  for each x of S (the variables  $id_x$  must be pairwise distinct). As a shorthand,  $\lfloor i \in [1,n]e_i/id_i \rfloor$  is noted  $\lfloor \frac{n}{i-1}e_i/id_i \rfloor$ .
- Universal quantification of a formula f(i) when i is in a given interval [1, n] is written f(i) (1 ≤ i ≤ n). This notation is straightforwardly extended to open and semi-open intervals.
- A deduction system is a set of rules written in the following way:

premise<sub>1</sub>...premise<sub>n</sub>
conclusion<sub>1</sub>...conclusion<sub>m</sub>

which can be read as "If all the premise; are true, then each conclusion; is true." The premises and conclusions are implicitly universally quantified over their free variables. If there are no premises, a single box is used.

 Kind, type and effect checking require a type and kind assignment function TK that is the mapping of variables to their type or kind. To distinguish whether TK is extended by a type or kind assignment, we respectively replace the → sign by : and :: . Kind, effect and type assertions are written in the following way:

$$TK \vdash d :: k$$

$$TK \vdash e : t ! f$$

These assertions mean that "TK proves that d has kind k, and e has type t and effect f." The empty assignment is noted  $\phi$ .

• FV(x) is the set of free variables and literals of the ordinary or description expression x.

#### 2. The FX-91 Kernel

The FX Kernel is a simple programming language that is the basis of the FX programming language. All of the constructs in the FX language can be directly explained by rewriting them into the simpler kernel language. Thus, the kernel forms the core of the FX language from the point of view of both the application programmer and the language implementor.

The FX Kernel has three language levels each with its own set of expressions: value expressions, description expressions and kind expressions. In the simplest terms, programs are value (or ordinary) expressions, types are descriptions, and kinds are the "types of types".

- Programs are written using value expressions. Value expressions form the lowest level of the language. Literals (e.g. #t) are examples of value expressions. It is possible to write sophisticated programs and only write value expressions.
- Declarations in value expressions are written using description expressions. Descriptions form the second level of the language. There are three kinds of descriptions: effect descriptions, type descriptions and description functions. As the name suggests, descriptions describe value expressions in particular, every legal value expression has both a type and an effect description. Most omitted declarations are reconstructed by FX.

Declarations in description expressions are written using kind expressions. Kinds form the third and highest level of the language. Kinds are the "types" of descriptions, and every legal description expression has a kind.

A complete specification for each level of the FX Kernel follows.

#### 2.1. Kinds

```
k := type \mid effect \mid (->> k_1 ...k_n)
```

For each kind special form, we give its syntax in its section header and provide an informal description of its usage. Kinds have neither static nor dynamic semantics.

#### 2.1.1. type

The kind expression type denotes the collection of descriptions that describe the values of computations (the so-called type expressions).

#### 2.1.2. effect

The kind expression effect denotes the collection of descriptions that describe the side-effects of computations (the so-called effect expressions).

2.1.3. 
$$(->> k_1...k_n)$$

A ->> expression denotes the collection of description functions that map descriptions of kind  $k_i$  to a type (the so-called type constructors).

#### 2.2. Descriptions

```
ti := id
       (di di_1...di_n)
       (-> ei ((id_1 ti_1)...(id_n ti_n)) ti_{n+1})
       (productof(id_1 ti_1)...(id_n ti_n))
       (\operatorname{sumof}(id_1 ti_1)...(id_n ti_n))
tx ::= (dx dx_1 ... dx_n) \mid
        (-> ei((id_1 tx_1)...(id_n tx_n)) tx_{n+1})
        (moduleof (abs ida_1 k_1)...(abs ida_n k_n)
                      (desc idd_1 dx_1)...(desc idd_p dx_p)
                      (val idv_1 tx_1)...(val idv_m tx_m))
        (\text{poly }((id_1 \ k_1)...(id_n \ k_n)) \ tx) \mid
        (productof(id_1 tx_1)...(id_n tx_n))
        (\operatorname{sumof}(id_1 tx_1)...(id_n tx_n))
ei := id \mid (maxeff ei_1...ei_n)
dx ::= tx \mid (dlambda ((id_1 k_1)...(id_n k_n)) tx) \mid di
di ::= ti \mid ei \mid id \mid (select e id)
```

The syntax of expressions e is given below.

Meta-variables that use i in their names (instead of x) denote description classes that can be omitted from user programs; they will be automatically inferred by the FX type and effect inference system. Such descriptions are said to be *inferable*. The class of inferable descriptions is contained in the class of descriptions.

The inclusion semantics is a reflexive and transitive deduction system based on the  $\Box$  partial order defined below. Intuitively, the description  $dx_1$  is included in  $dx_2$  (noted  $dx_1 \Box dx_2$ ) iff  $dx_1$  is more constrained than  $dx_2$ . We note  $dx_1 \sim dx_2$  if  $dx_1 \Box dx_2$  and  $dx_2 \Box dx_1$ .

For each description special form, we give its syntax in its section header and provide an informal description of its usage, its static semantics and its inclusion semantics (if any). There is no dynamic semantics for descriptions.

#### 2.2.1. id

A variable denotes the description to which it is bound.

There are seven constant identifiers. fx..unit is the type of expressions used only for their side-effects. fx..bool is the type of booleans. fx..pure, fx..read, fx..write and fx..init are the effects of expressions that are respectively referentially transparent, read-only, write-only and allocation-only. (fx..refof t) is the type of mutable references to values of type t. They are defined in the fx module (see Chapter 3) to limit the number of reserved identifiers.

#### Static Semantics

#### 2.2.2. $(dx_0 dx_1...dx_n)$

A description application is the type obtained by applying the type constructor  $dz_0$  to the descriptions  $dz_i$ .

#### Static Semantics

$$TK \vdash dx_0 :: (->> k_1...k_n)$$

$$TK \vdash dx_i :: k_i \quad (1 \le i \le n)$$

$$TK \vdash (dx_0 \ dx_1...dx_n) :: type$$

Inclusion Semantics

$$((dlambda ((id_1 k_1)...(id_n k_n)) tx) dx_1...dx_n) \sim \\ [n]_{i=1} dx_i/id_i]tx$$

$$id_i \notin FV(tx) \quad (1 \le i \le n)$$

$$(dlambda ((id_1 k_1)...(id_n k_n)) (tx id_1...id_n)) \sim tx$$

2.2.3. (-> 
$$ei((id_1 tx_1)...(id_n tx_n)) tx_{n+1})$$

The id; must be distinct.

An  $\rightarrow$  expression is the type of subroutines that map values of type  $tz_i$  to a value of type  $tz_{n+1}$  while performing the side-effect ei. An  $\rightarrow$  expression is a binding construct.

Static Semantics

Inclusion Semantics

$$id'_{i} \notin \bigcup_{j=1}^{n+1} FV(tx_{j}) \quad (1 \leq i \leq n)$$

$$(-) ei ((id_{1} tx_{1})...(id_{n} tx_{n})) tx_{n+1})$$

$$\sim$$

$$(-) ei ((id'_{1} tx_{1})...(id'_{n} [_{j=1}^{n-1} id'_{j}/id_{j}]tx_{n}))$$

$$[_{i=1}^{n} id'_{j}/id_{j}]tx_{n+1})$$

2.2.4. (dlambda 
$$((id_1 k_1)...(id_n k_n)) tx$$
)

The id, must be distinct.

A dlambda expression is the type constructor that maps descriptions of kinds  $k_i$  to the type tx. A dlambda expression is a binding construct.

$$TK[\underset{i=1}{n}id_i :: k_i] \vdash tx :: type$$

$$TK \vdash (dlambda ((id_1 k_1)...(id_n k_n)) tx)$$

$$:: (->> k_1...k_n)$$

Inclusion Semantics

$$id'_{i} \notin FV(tx) \quad (1 \leq i \leq n)$$

$$(dlambda ((id_{1} k_{1})...(id_{n} k_{n})) tx)$$

$$\sim$$

$$(dlambda ((id'_{1} k_{1})...(id'_{n} k_{n})) \begin{bmatrix} n \\ i=1 id'_{i}/id_{i} \end{bmatrix} tx)$$

#### 2.2.5. (maxeff $ei_1...ei_n$ )

A maxeff expression is the cumulative effect of the effects  $ei_i$ .

Static Semantics

Inclusion Semantics

$$(\text{maxeff } ei_1 \ ei_2) \sim (\text{maxeff } ei_2 \ ei_1)$$

2.2.6. (moduleof (abs  $ida_1 \ k_1$ )...(abs  $ida_1 \ k_n$ )

(desc  $idd_1 \ dx_1$ )...(desc  $idd_p \ dx_p$ )

(val  $idv_1 \ tx_1$ )...(val  $idv_m \ tx_m$ ))

The ida, iddk and idv; must be distinct.

A module of expression is the type of modules that export the abstract descriptions  $ida_i$ , the transparent descriptions  $idd_k$  and the values  $idv_j$ . A module of expression is a binding construct.

Static Semantics

$$TK[_{i=1}^{n} ida_{i} :: k_{i}] \vdash dz_{k} :: kd_{k} \quad (1 \leq k \leq p)$$

$$TK[_{i=1}^{n} ida_{i} :: k_{i}] \vdash [_{k=1}^{p} dz_{k}/idd_{k}] tz_{j} :: type$$

$$(1 \leq j \leq m)$$

$$TK \vdash (moduleof \quad (abs ida_{1} k_{1}) ... (abs ida_{n} k_{n}) \quad (desc idd_{1} dz_{1}) ... (desc idd_{p} dz_{p}) \quad (val idv_{1} tz_{1}) ... (val idv_{m} tz_{m})) :: type$$

#### **Inclusion Semantics**

$$\begin{array}{c} \pi, \ \sigma, \ \tau \ \text{are permutations on} \ [1,n], \ [1,p], \ [1,m] \\ & (\text{moduleof} \\ & (\text{abs} \ ida_1 \ k_1) ... (\text{abs} \ ida_n \ k_n) \\ & (\text{desc} \ idd_1 \ dx_1) ... (\text{desc} \ idd_p \ dx_p) \\ & (\text{val} \ idv_1 \ tx_1) ... (\text{val} \ idv_m \ tx_m)) \\ & \sim \\ & (\text{moduleof} \\ & (\text{abs} \ ida_{\pi(1)} \ k_{\pi(1)}) ... (\text{abs} \ ida_{\pi(n)} \ k_{\pi(n)}) \\ & (\text{desc} \ idd_{\sigma(1)} \ dx_{\sigma(1)}) ... (\text{desc} \ idd_{\sigma(p)} \ dx_{\sigma(p)}) \\ & (\text{val} \ idv_{\tau(1)} \ tx_{\tau(1)}) ... (\text{val} \ idv_{\tau(m)} \ tx_{\tau(m)})) \end{array}$$

$$dx_i \quad \Box \quad dx'_i \quad (1 \leq i \leq p')$$

$$tx_j \quad \Box \quad tx'_j \quad (1 \leq j \leq m')$$

$$n \text{ (resp. } m, p) \geq n' \text{ (resp. } m', p')$$

$$(\text{moduleof} \quad (\text{abs } ida_1 \ k_1) \dots (\text{abs } ida_n \ k_n)$$

$$(\text{desc } idd_1 \ dx_1) \dots (\text{desc } idd_p \ dx_p)$$

$$(\text{val } idv_1 \ tx_1) \dots (\text{val } idv_m \ tx_m))$$

$$\Box$$

$$(\text{moduleof} \quad (\text{abs } ida_1 \ k_1) \dots (\text{abs } ida_n \ k_{n'})$$

$$(\text{desc } idd_1 \ dx'_1) \dots (\text{desc } idd_p' \ dx'_p')$$

$$(\text{val } idv_1 \ tx'_1) \dots (\text{val } idv_{m'} \ tx'_{m'}))$$

#### 2.2.7. (poly $((id_1 k_1)...(id_n k_n)) tx$ )

The id, must be distinct.

A poly expression is the type of polymorphic expressions abstracted over descriptions of kind  $k_i$ . A poly expression is a binding construct.

Static Semantics

$$TK[_{i=1}^{n}id_{i}::k_{i}]\vdash tx:: type$$

$$TK\vdash (poly ((id_{1}k_{1})...(id_{n}k_{n})) tx):: type$$

Inclusion Semantics

$$tx \sqsubset tx'$$

$$(poly ((id_1 k_1)...(id_n k_n)) tx)$$

$$\sqsubseteq$$

$$(poly ((id_1 k_1)...(id_n k_n)) tx')$$

$$id'_i \notin FV(tx) \quad (1 \le i \le n)$$

$$(\text{poly } ((id_1 \ k_1)...(id_n \ k_n)) \ tx)$$

$$\sim$$

$$(\text{poly } ((id'_1 \ k_1)...(id'_n \ k_n)) \left[ \prod_{i=1}^n id'_i/id_i \right] tx)$$

#### **2.2.8.** (productof $(id_1 \ tx_1)...(id_n \ tx_n)$ )

The id; must be distinct.

A productof expression is the type of aggregate values with named fields. Each field  $id_i$  corresponds to a value of type  $tx_i$ .

$$TK \vdash tx_i :: type \quad (1 \le i \le n)$$

$$TK \vdash (productof (id_1 tx_1)...(id_n tx_n)) :: type$$

#### Inclusion Semantics

$$\begin{array}{c|cccc} tx_i & \sqsubset tx'_i & (1 \leq i \leq m) \\ n \geq m & \\ \hline & (\text{productof} & (id_1 & tx_1) ... (id_n & tx_n)) \\ \hline & \sqsubset & \\ & (\text{productof} & (id_1 & tx'_1) ... (id_m & tx'_m)) & \\ \end{array}$$

#### 2.2.9. (select e id)

A select expression is the description named id, either abstract or transparent, that is exported by the module e. The effect of e must be pure to prevent type abstraction violation.

#### Static Semantics

$$TK \vdash e : (moduleof \\ (abs ida_1 k_1)...(abs ida_n k_n) \\ (desc idd_1 dx_1)... \\ (val idv_1 tx_1)...) \\ ! fx..pure$$

$$TK \vdash (select e ida_i) :: k_i \quad (1 \le i \le n)$$

$$TK \vdash e : (\text{moduleof} \\ (\text{abs } ida_1 \ k_1) ... (\text{abs } ida_n \ k_n) \\ (\text{desc } idd_1 \ dx_1) ... (\text{desc } idd_m \ dx_m) \\ (\text{val } idv_1 \ tx_1) ...) : \text{fx.pure} \\ TK[^n_{i=1}ida_i :: k_i] \vdash dx_j :: kd_j \ (1 \leq j \leq m) \\ TK \vdash (\text{select } e \ idd_j) :: kd_j \ (1 \leq j \leq m)$$

#### Inclusion Semantics

$$TK \vdash e : (\text{moduleof} \\ (\text{abs } ida_1 \ k_1) ... (\text{abs } ida_n \ k_n) \\ (\text{desc } idd_1 \ dx_1) ... (\text{desc } idd_m \ dx_m) \\ (\text{val } idv_1 \ tx_1) ...) \\ (\text{select } e \ idd_i) \\ \sim \\ [_{j=1}^n (\text{select } e \ ida_j) / ida_j] dx_i \quad (1 \leq i \leq m)$$

#### 2.2.10. (sum of $(id_1 tx_1)...(id_n tx_n)$ )

The id; must be distinct.

A sum of expression is the type of tagged values of type  $tx_i$ with tag idi.

#### Static Semantics

$$TK \vdash tz_i :: type \quad (1 \le i \le n)$$

$$TK \vdash (sumof (id_1 tz_1)...(id_n tz_n)) :: type$$

#### Inclusion Semantics

$$\pi$$
 is a permutation on  $[1, n]$ 

$$(sumof (id_1 tx_1)...(id_n tx_n))$$
 $\sim$ 

$$(sumof (id_{\pi(1)} tx_{\pi(1)})...(id_{\pi(n)} tx_{\pi(n)}))$$

#### Values 2.3.

```
e ::= literal |
      sugar
      id |
       (e0 e1...en) |
       (begin e_0 e_1...e_n)
       (extend e_0 e_1)
       (extract tx e id) |
       (if e0 e1 e2) |
       (lambda ([id_1 | (id'_1 tx'_1)]...[id_n | (id'_n tx'_n)]) e) |
       (1et ((id_1 e_1)...(id_n e_n)) e)
       (load literal)
       (module (define-abstraction ida<sub>1</sub> ka<sub>1</sub> dxa<sub>1</sub>)...
                   (define-abstraction ida_n ka_n dxa_n)
                 (define-description idd_1 \ dxd_1)...
                    (define-description idd_m dxd_m)
                 (define idv_1 e_1)...(define idv_p e_p)
                 (define-typed idt_1 tx_1 e'_1)...
                    (define-typed idt_q tx_q e'_q))
       (open e)
       (plambda ((id_1 k_1)...(id_n k_n)) e)
       (product tx e_1...e_n)
       (\text{proj } e dx_1...dx_n)
       (sum tx id e) |
       (tagcase tx e id e_1 e_2)
       (the tx e)
       (with e_0 e_1)
```

For each expression special form (see also the Sugars section), we give its syntax in its section header and provide an informal description of its usage, its static semantics and its dynamic semantics.

The static semantics of expressions is defined modulo the inclusion semantics of descriptions:

TK	-	e: tx! ei
ei	~	ei
tx	~	tx'
TK	+ e	: tx ! e?

The dynamic semantics is a deduction system based on the transitively closed - relation defined over pairs made of values v or expressions e, and stores  $\sigma$ . A value is either a literal, or a list of values or expressions in brackets  $\langle v_1...v_n \rangle$ . Stores are functions that map locations to values.

#### 2.3.1. Literals

There are three kernel literals: #t and #f for the fx..bool type and #u for the fx..unit type. Other literals are introduced via the fx module. A literal evaluates to itself and is a pure expression. All fx literals are immutable.

```
TK ⊢ #t:fx..bool!fx..pure
TK ⊢ #f:fx..bool!fx..pure
TK \vdash #u: fx..unit! fx..pure
```

#### Dynamic Semantics

A literal expression evaluates to itself.

#### 2.3.2. id

A variable denotes the value it is bound to.

There are three constant identifiers: the fx.ref subroutine allocates and returns a new reference with initial value val0, the fx. subroutine returns the value stored in ref and the fx.:= subroutine replaces the value stored in ref with val1 and returns #u.

#### Static Semantics

```
TK[id:tx] \vdash id:
                     tx ! fx..pure
  TK \vdash fx.ref
                     (poly ((t type))
                       (-> fx..init
                            ((val0 t))
                            (fx..refof t)))
    TK \vdash fx.^{-}:
                     (poly ((t type))
                       (-> fx . . read
                            ((ref (fx..refof t)))
                            t))
   TK \vdash fx. := :
                     (poly ((t type))
                       (-> fx..write
                            ((ref (fx..refof t))
                             (val1 t))
                            fx..unit))
```

#### **Dynamic Semantics**

 $(\#u, \sigma[l \mapsto v])$ 

#### 2.3.3. $(e_0 e_1...e_n)$

The expressions  $e_i$  are successively evaluated to values  $v_i$  and the value resulting from applying  $v_0$  (after implicit projection if necessary, see open below) to  $v_i$  is returned.

#### Static Semantics

$$TK \vdash e_0 : (-> ei ((id_1 tx_1)...(id_n tx_n)) tx_{n+1})$$

$$! ei_0$$

$$tx'_i \sim \begin{bmatrix} i-1 \\ j-1 ej/id_j \end{bmatrix} tx_i \quad (1 \le i \le n+1)$$

$$TK \vdash tx'_i :: type \quad (1 \le i \le n+1)$$

$$TK \vdash e_i : tx'_i ! ei_i \quad (1 \le i \le n)$$

$$TK \vdash (e_0 e_1...e_n) : tx'_{n+1}$$

$$! (maxeff ei ei_0 ei_1...ei_n)$$

#### Dynamic Semantics

$$(e_{i}, \sigma) \rightarrow (v_{i}, \sigma')$$

$$((v_{0}...v_{i-1} e_{i}...e_{n}), \sigma) \rightarrow ((v_{0}...v_{i} e_{i+1}...e_{n}), \sigma')$$

$$((\langle *lambda* (id_{1}...id_{n}) e \rangle v_{1}...v_{n}), \sigma)$$

$$\rightarrow ([^{n}_{i=1}v_{i}/id_{i}]e, \sigma)$$

#### 2.3.4. (begin $e_0 e_1...e_n$ )

The expressions  $e_i$  are successively evaluated to values  $v_i$  and  $v_n$  is returned.

#### Static Semantics

#### Dynamic Semantics

$$(e,\sigma) \rightarrow (v,\sigma')$$

$$((\text{begin } e),\sigma) \rightarrow (v,\sigma')$$

$$((e_0,\sigma) \rightarrow (v_0,\sigma')$$

$$((\text{begin } e_0 \ e_1 \ e_2...),\sigma)$$

$$\rightarrow$$

$$((\text{begin } e_1 \ e_2...),\sigma')$$

#### 2.3.5. (extend $e_0$ $e_1$ )

The expression  $e_0$  is evaluated to a module  $v_0$  and the value of module  $e_1$ , extended with all the bindings introduced by  $v_0$ , is returned. The expression  $e_1$  has access to the bindings of  $e_0$ . In the case of conflict, the bindings of  $v_1$  take precedence. An extend expression is a binding construct.

```
TK \vdash e_0 : (module of (abs ida_{01} k_{01}) ... (abs ida_{0n_0} k_{0n_0}) \\ (abs ida_{01} k_{01}) ... (abs ida_{0n_0} k_{0n_0}) \\ (val idv_{01} tz_{01}) ... (val idv_{0p_0} tz_{0p_0})) \\ ! ei_0 \\ TK \vdash (with e_0 e_1) : tx ! ei \\ tx \sim (module of (abs ida_{11} k_{11}) ... (abs ida_{1n_1} k_{1n_1}) \\ (aesc idd_{11} dz_{11}) ... (aesc idd_{1m_1} dz_{1m_1}) \\ (val idv_{11} tz_{11}) ... (val idv_{1p_1} tz_{1p_1})) \\ \{ i_{n_1}^{n_2} ida_{2i} \} = \{ i_{n_1}^{n_1} ida_{0i} \} - \{ i_{n_1}^{n_1} ida_{1i} \} \\ \{ i_{n_1}^{n_2} idv_{2i} \} = \{ i_{n_1}^{n_1} idv_{0i} \} - \{ i_{n_1}^{n_1} idv_{1i} \} \\ \{ i_{n_1}^{n_2} idv_{2i} \} = \{ i_{n_1}^{n_2} idv_{0i} \} - \{ i_{n_1}^{n_1} idv_{1i} \} \\ TK \vdash (extend e_0 e_1) \\ : (module of (abs ida_{21} k_{21}) ... (abs ida_{2n_2} k_{2n_2}) \\ (abs ida_{21} k_{21}) ... (abs ida_{1n_1} k_{1n_1}) \\ (aesc idd_{21} dz_{21}) ... (aesc idd_{2m_2} dz_{2m_2}) \\ (aesc idd_{11} dz_{11}) ... (aesc idd_{1m_1} dz_{1m_1}) \\ (val idv_{21} tz_{21}) ... (val idv_{2p_2} tz_{2p_2}) \\ (val idv_{11} tz_{11}) ... (val idv_{1p_1} tz_{1p_1})) \\ ! (maxeff ei_0 ei)
```

#### **Dynamic Semantics**

An \*extend\* expression is a special form only available in the dynamic semantics.

$$(e_0,\sigma) \rightarrow (v_0,\sigma')$$

$$((\texttt{extend}\ e_0\ e_1),\sigma) \rightarrow ((\texttt{*extend*}\ v_0\ (\texttt{with}\ v_0\ e_1)),\sigma')$$

$$\begin{array}{c} ((\texttt{*extend*}\ \langle \texttt{*module*}\ (id_1\ v_1)...(id_n\ v_n)\rangle \\ & (\texttt{*module*}\ (id'_1\ v'_1)...(id'_m\ v'_m)\rangle),\sigma) \\ & \rightarrow \\ & ((\texttt{*module*}\ (id''_1\ v_1)...(id''_p\ v_p) \\ & (id'_1\ v'_1)...(id'_m\ v'_m)\rangle,\sigma) \end{array}$$

where 
$$\{_{k=1}^{p} id''_{k}\} = \{_{k=1}^{n} id_{k}\} - \{_{k=1}^{m} id'_{k}\}.$$

#### 2.3.6. (extract tx e id)

The expression e of product type tx is evaluated to an aggregate value v. The value of the field id of v is returned.

Static Semantics

$$TK \vdash tx :: type$$

$$TK \vdash e : tx ! ei$$

$$tx \sim (productof (id_1 tx_1)...(id_n tx_n))$$

$$TK \vdash (extract tx e id_i) : tx_i ! ei$$

#### Dynamic Semantics

$$(e,\sigma) \rightarrow (v,\sigma')$$

$$((\text{extract } tx \ e \ id),\sigma)$$

$$\rightarrow$$

$$((\text{extract } tx \ v \ id),\sigma')$$

$$\begin{array}{c} ((\texttt{extract}\ tx\ \langle *\texttt{product}*\ (id_1\ v_1)...(id_n\ v_n)\rangle\ id_i),\sigma) \\ \longrightarrow \\ (v_i,\sigma) \end{array}$$

#### 2.3.7. (if $e_0 \ e_1 \ e_2$ )

An if expression evaluates  $e_0$  to the value  $v_0$ . If  $v_0$  is #t (resp. #1), then the value of  $e_1$  (resp.  $e_2$ ) is returned.

Static Semantics

$$TK \vdash e_0 : fx..bool ! ei_0$$
 $TK \vdash e_1 : tx ! ei_1$ 
 $TK \vdash e_2 : tx ! ei_2$ 
 $TK \vdash (if e_0 e_1 e_2) : tx ! (maxeff ei_0 ei_1 ei_2)$ 

Dynamic Semantics

2.3.8. (lambda (
$$[id_1 \mid (id'_1 tx'_1)]...[id_n \mid (id'_n tx'_n)]) e$$
)

The  $id_i$  and  $id'_i$  must be distinct.

A lambda expression denotes the subroutine that, when applied to n values  $v_i$ , returns the value of e with the argument values  $v_i$  substituted for the formals  $id_i$  and  $id'_i$ . A lambda expression is a binding construct.

Static Semantics

$$TK_{i} = TK_{j=1}^{n-1} id_{j} : tx_{j}^{\prime}] \quad (1 \leq i \leq n+1)$$

$$TK_{i} \vdash ti_{i} :: type \quad (1 \leq i \leq n)$$

$$TK_{i} \vdash tx_{i}^{\prime} :: type \quad (1 \leq i \leq n)$$

$$TK_{n+1}[_{i=1}^{n} id_{i} : ti_{i}] \vdash e : tx ! ei$$

$$TK \vdash (lambda ([id_{1} \mid (id_{1} \mid tx_{1}^{\prime})]...[id_{n} \mid (id_{n} \mid tx_{n}^{\prime})])$$

$$e)$$

$$: (-> ei ([(id_{1} \mid ti_{1}) \mid (id_{1} \mid tx_{1}^{\prime})]...$$

$$[(id_{n} \mid ti_{n}) \mid (id_{n} \mid tx_{n}^{\prime})]) \quad tx)$$

$$! \text{ fx..pure}$$

Dynamic Semantics

2.3.9. (let 
$$((id_1 e_1)...(id_n e_n)) e$$
)

A let expression simultaneously binds each  $id_i$  to the value  $v_i$  of  $e_i$ . The value of e, evaluated in an augmented environment that binds  $id_i$  to  $v_i$ , is returned. A let expression is a binding construct.

Static Semantics

$$TK \vdash e_i : tx_i ! ei_i (1 \le i \le n)$$

$$G = \{i | not\_expansive(e_i) (1 \le i \le n)\}$$

$$TK[_{i \in [1,n]\_G} id_i : tx_i] \vdash [_{i \in G} e_i / id_i] e : tx ! ei$$

$$TK \vdash [_{i=1}^n e_i / id_i] tx :: type$$

$$TK \vdash ((id_1 e_1) ... (id_n e_n)) e)$$

$$: [_{i=1}^n e_i / id_i] tx$$

$$! (maxeff ei_1 ... ei_n ei)$$

where an expression is *not\_expansive* iff it is a literal, an identifier, a lambda expression, a plambda expression or a non-application compound expression for which each value subexpression is *not\_expansive*.

Dynamic Semantics

$$((\operatorname{let}((id_1 \ e_1)...(id_n \ e_n)) \ e), \sigma) \rightarrow \\ ((\langle *\operatorname{lambda} * (id_1...id_n) \ e \rangle \ e_1...e_n), \sigma)$$

#### 2.3.10. (load literal)

The expression in the file named *literal* is produced as a value. No free variables are allowed in a load file, except if defined in the fx module (see next chapter).

Static Semantics

```
\phi \vdash (\text{with fx (include literal})) : tx ! ei
TK \vdash (\text{load literal}) : tx ! ei
```

where include is an implementation-specific function that returns the expression in the file whose name is given as an argument.

Dynamic Semantics

$$\frac{((\text{with fx (include literal)}), \sigma) \rightarrow (v, \sigma')}{((\text{load literal}), \sigma) \rightarrow (v, \sigma')}$$

2.3.11.

(module (define-abstraction 
$$ida_1 \ k_1 \ dxa_1$$
)...

(define-abstraction  $ida_n \ k_n \ dxa_n$ )

(define-description  $idd_1 \ dxd_1$ )...

(define-description  $idd_m \ dxd_m$ )

(define  $idv_1 \ e_1$ )...(define  $idv_p \ e_p$ )

(define-typed  $idt_1 \ tx_1 \ e'_1$ )...

(define-typed  $idt_q \ tx_q \ e'_q$ ))

The ida, idd, idvk, idt, must be distinct.

A module expression evaluates to a module that contains the abstract descriptions  $ida_i$ , the transparent descriptions  $idd_j$  and the values of  $idv_k$  and  $idt_l$ . The representation descriptions  $dxa_i$  of  $ida_i$  can be mutually recursive. The values  $e_k$  and  $e'_l$  are successively evaluated and can be mutually recursive. For each non-effect abstract description  $ida_i$ , two subroutines are automatically defined in the scope of the module expression:  $up-ida_i$  maps from the representation description to the abstract description, while  $dz=n-ida_i$  goes the opposite way.

#### Static Semantics

```
\overline{TK[_{i=1}^{n}ida_{i}::k_{i}]}
                 TK_1[_{k=1}^p idv_k : ti_k][_{l=1}^q idt_l : [_{j=1}^m dxd_j/idd_j]tx_l]
                         [_{i=1}^{n} up - ida_{i} : Up(ida_{i}, k_{i}, dxa_{i})]
                          \begin{bmatrix} n \\ i=1 \end{bmatrix} down - ida_i : Down(ida_i, k_i, dxa_i) \end{bmatrix}
TK_1 \vdash
                 dxa_i :: k_i \quad (1 \leq i \leq n)
TK_1
                  dxd_j :: kd_j \quad (1 \le j \le m)
TK_1
                  \begin{bmatrix} m \\ j=1 \end{bmatrix} dxd_j / idd_j \end{bmatrix} tx_l :: type \quad (1 \le l \le q)
                  \begin{bmatrix} m \\ j=1 \end{bmatrix} dx d_j / i dd_j e_k : ti_k ! ei_k \quad (1 \le k \le p)
TK_2
                  \begin{bmatrix} m \\ j=1 & dxd_j / idd_j \end{bmatrix} e'_l : tx_l ! ei'_l \quad (1 \le l \le q)
TK_2
         ⊢
      TK \vdash (module \\ (define-abstraction ida_1 k_1 dxa_1)...
                  (define-abstraction ida_n k_n dxa_n)
(define-description idd_1 dxd_1)...
                         (define-description idd_m dxd_m)
                  (define idv_1 e_1)...(define idv_p e_p)
                  (define-typed idt_1 tx_1 e'_1).
                         (define-typed idt_q tx_q e'_q))
                 : (moduleof
                        (abs ida_1 \ k_1)...(abs ida_n \ k_n)

desc idd_1 \ dxd_1)...(desc idd_m \ dxd_m)

val idv_1 \ ti_1)...(val idv_p \ ti_p)
                       (val idt_1 tx_1)...(val idt_q tx_q))
                 ! (maxeff ei1...eip ei1...eiq)
```

with the following definitions (where id, are fresh):

```
\begin{array}{l} Up(d_1, \mathsf{type}, d_2) = (-) \, \mathtt{fx..pure} \, (d_2) \, d_1) \\ Up(d_1, (-) k_1 ... k_n), d_2) = \\ (\mathsf{poly} \, ((id_1 \, k_1) ... (id_n \, k_n)) \\ Up((d_1 \, id_1 ... id_n), \mathsf{type}, (d_2 \, id_1 ... id_n))) \\ Down(d_1, k, d_2) = Up(d_2, k, d_1) \end{array}
```

#### Dynamic Semantics

The \*module-no-rec\* and \*rec\* expressions are special forms only available in the dynamic semantics.

```
((module
           (define-abstraction ida_1 \ k_1 \ dxa_1)...
               (define-abstraction ida_n k_n dxa_n)
           (define-description idd_1 \ dxd_1)...
               (define-description idd_m \ dxd_m)
           (define idv_1 e_1)...(define idv_p e_p)
           (define-typed idt_1 tx_1 e'_1)...
               (define-typed idt_q tx_q e'_q)),\sigma)
((*module-no-rec*
      (idv_1 \begin{bmatrix} n \\ i=1 \end{bmatrix} (lambda (id) id) / up - ida_i]
                \binom{n}{i=1}(\text{lambda}(id) id)/\text{down}-ida_i]e_1)...
       (idv_p \begin{bmatrix} n \\ i=1 \end{bmatrix} (lambda (id) id) / up - ida_i]
                \begin{bmatrix} n \\ i=1 \end{bmatrix} (lambda (id) id)/down-ida; e_p)
       (idt_1 \begin{bmatrix} n \\ i=1 \end{bmatrix} (lambda (id) id) / up - ida_i]
                \begin{bmatrix} n \\ i=1 \end{bmatrix} (lambda (id) id)/down-ida<sub>i</sub>]e'_1)...
       (idt_q \begin{bmatrix} n \\ i=1 \end{bmatrix} (lambda (id) id) / up - ida_i]
                \begin{bmatrix} n \\ i=1 \end{bmatrix} (lambda (id) id) / down - ida_i \end{bmatrix} e'_{\theta}), \sigma
```

$$(\{*\mathtt{rec*}\ ((\mathit{id}_1\ v_1)...(\mathit{id}_p\ v_p))\ \mathit{id}_k\},\sigma) \rightarrow \\ ([\begin{smallmatrix} p \\ i=1 \end{smallmatrix} ( *\mathtt{rec*}\ ((\mathit{id}_1\ v_1)...(\mathit{id}_p\ v_p))\ \mathit{id}_i \rangle / \mathit{id}_i]v_k,\sigma)$$

#### 2.3.12. (open e)

An open expression returns, from the polymorphic expression e, the value of e with the polymorphic description variables  $id_i$  of e replaced by inferred description expressions  $di_i$ . When a polymorphic value is directly applied to values, open is used to perform implicit projection.

#### Static Semantics

$$TK \vdash e: (poly ((id_1 k_1)...(id_n k_n)) tx) ! ei$$

$$TK \vdash di_i :: k_i \quad (1 \le i \le n)$$

$$TK \vdash (open e) : [_{i=1}^n di_i / id_i] tx ! ei$$

$$TK \vdash e_0 : (poly ((id_1 \ k_1)...(id_m \ k_m)) \ tx') ! \ ei'$$

$$TK \vdash ((open \ e_0) \ e_1...e_n) : tx ! \ ei$$

$$TK \vdash (e_0 \ e_1...e_n) : tx ! \ ei$$

#### Dynamic Semantics

$$((open e), \sigma) \rightarrow (e, \sigma)$$

#### 2.3.13. (plambda $((id_1 k_1)...(id_n k_n)) e$ )

The id; must be distinct.

A plambda expression denotes the polymorphic value that, when projected onto n description expressions  $dx_i$  of kind  $k_i$ , returns the value of the pure expression e with the argument values  $dx_i$  substituted for the formals  $id_i$ . A plambda expression is a binding construct.

#### Static Semantics

#### Dynamic Semantics

$$((\text{plambda}((id_1 k_1)...) e), \sigma) \rightarrow (e, \sigma)$$

#### 2.3.14. (product $tx e_1...e_n$ )

The n expressions  $e_i$  are successively evaluated to values  $v_i$ . A product expression evaluates to an aggregate value of product type tz, with each field  $id_i$  having the value  $v_i$ .

#### Static Semantics

$$TK \vdash tx :: type$$

$$tx \sim (productof (id_1 tx_1)...(id_n tx_n))$$

$$TK \vdash e_i : tx_i ! ei_i (1 \le i \le n)$$

$$TK \vdash (product tx e_1...e_n) : tx$$

$$! (maxeff ei_1...ei_n)$$

#### Dynamic Semantics

$$((\text{product } tx \ e_1...e_n), \sigma) \\ \rightarrow \\ (((\text{lambda } (id'_1...id'_n) \ (*\text{product*} \ (id_1 \ id'_1)...(id_n \ id'_n)))) \\ e_1...e_n), \sigma)$$

where the  $id_i$  are fresh.

#### 2.3.15. (proj $e dx_1...dx_n$ )

A proj expression projects the polymorphic expression e onto the description expressions  $dx_i$ , returning the corresponding value.

#### Static Semantics

$$TK \vdash e: (poly ((id_1 k_1)...(id_n k_n)) tx) ! ei$$

$$TK \vdash dx_i :: k_i \quad (1 \le i \le n)$$

$$TK \vdash (proj e dx_1...dx_n) : \begin{bmatrix} n \\ i=1 dx_i / id_i \end{bmatrix} tx ! ei$$

#### **Dynamic Semantics**

$$((\text{proj } e \ d_1...), \sigma) \rightarrow (e, \sigma)$$

#### 2.3.16. (sum tx id e)

The expression e is evaluated to v and a tagged value of sum type tx with tag id and value v is returned.

#### Static Semantics

$$TK \vdash tx :: type$$
 $tx \sim (sum of (id tx)...)$ 
 $TK \vdash e : tx ! ei$ 
 $TK \vdash (sum tx id e) : tx ! ei$ 

#### Dynamic Semantics

#### 2.3.17. (tagcase $tx e id e_1 e_2$ )

The expressions e,  $e_1$  and  $e_2$  are successively evaluated to values v,  $v_1$  and  $v_2$ . The value v is a tagged value of type tx with tag  $id_j$  and value v'. If id is  $id_j$ , then the result of applying  $v_1$  to v' is returned, otherwise the result of applying  $v_2$  to v.

#### Static Semantics

#### Dynamic Semantics

```
((tagcase tx \ e \ id \ e_1 \ e_2),\sigma)

\rightarrow
(((lambda (id_1 \ id_2 \ id_3)
(tagcase tx \ id_1 \ id \ id_2 \ id_3)) e \ e_1 \ e_2),\sigma)
```

where id; are fresh.

$$((\text{tagcase } tx \ (*\text{sum*} \ id \ v) \ id \ v_1 \ v_2), \sigma) \rightarrow ((v_1 \ v), \sigma)$$

$$((\text{tagcase } tx \ (*\text{sum*} id \ v) \ id \ v_1 \ v_2), \sigma) \rightarrow \\ ((v_2 \ (*\text{sum*} id \ v)), \sigma)$$

#### 2.3.18. (the tx e)

The type of e must be included in tx. The value of e is returned.

Static Semantics

$$TK \vdash tx :: type$$
 $TK \vdash e : tx' ! ei$ 
 $tx' \sqsubseteq tx$ 
 $TK \vdash (the tx e) : tx ! ei$ 

Dynamic Semantics

$$((the tx e), \sigma) \rightarrow (e, \sigma)$$

#### 2.3.19. (with $e_0 e_1$ )

The pure expression  $e_0$  is evaluated to  $v_0$  and the value of  $e_1$ , evaluated in an environment extended with all the bindings defined in the module  $v_0$ , is returned. A with expression is a binding construct.

#### Static Semantics

```
TK \vdash e_0 : (\texttt{moduleof} \\ (\texttt{abs} ida_1 \ k_1) ... (\texttt{abs} ida_n \ k_n) \\ (\texttt{desc} idd_1 \ dx_1) ... (\texttt{desc} idd_m \ dx_m) \\ (\texttt{val} idv_1 \ tx_1) ... (\texttt{val} idv_p \ tx_p)) \\ : \texttt{fx..pure} \\ \theta = \begin{bmatrix} p \\ k=1 \\ [*] (\texttt{with} \ e_0 \ idv_k) / idv_k \end{bmatrix} \\ \begin{bmatrix} n \\ i=1 \\ [*] (\texttt{select} \ e_0 \ ida_i) / ida_i \end{bmatrix} \\ \begin{bmatrix} n \\ j=1 \\ [*] dx_j / idd_j \end{bmatrix}
TK \vdash (\texttt{with} \ e_0 \ idv_k) : \theta tx_k \ ! \ \texttt{fx..pure} \quad (1 \le k \le p)
```

```
TK \vdash e_0 : (moduleof (abs ida_1 k_1)...(abs ida_n k_n) (desc idd_1 dx_1)...(desc idd_m dx_m) (val idv_1 x_1)...(val idv_p tx_p))
! fx..pure
\theta = \begin{bmatrix} p \\ k=1 \text{ (with } e_0 idv_k) / idv_k \end{bmatrix}
\begin{bmatrix} \frac{n}{j=1} dx_j / idd_j \end{bmatrix}
TK \vdash \theta e_1 : tx ! e_i
TK \vdash (with e_0 e_1) : \theta tx ! \theta e_i
```

#### Dynamic Semantics

((with (\*module\* 
$$(idv_1 \ v_1)...(idv_p \ v_p)) \ e), \sigma$$
)
$$([\stackrel{p}{i=1}v_i/idv_i]e, \sigma)$$

#### 2.4. Sugars

```
sugar ::= (and e_1...e_n)
            (cond (e_1 e'_1)...(e_n e'_n) (else e'_{n+1}))
            (let*((id_1 e_1)...(id_n e_n)) e)
            (letrec ((id_1 e_1)...(id_n e_n)) e) |
            (\text{match } e \ (pat_1 \ e_1)...(pat_n \ e_n))
            (or e_1...e_n)
            id_1.id_2...id_n.id
            id_1..id_2
            [e dx_1...dx_n]
            (define head e) |
            (define-datatype [(id (id_1 k_1)...(id_n k_n)) | id]
                   (id'_1 dx_{11} ... dx_{1m_1})...
                   (id_p dx_{p1} ... dx_{pm_p}))
            (do (id e0 e;) (e, e,) e) |
            (abs (id_1 id_2...id_n) k)
            (val (id_1 id_2...id_n) tx)
```

$$pat ::= |literal| - | id | (e pat_1...pat_n)$$

head ::= 
$$id \mid (head (id_1 tx_1)...(id_n tx_n)) \mid [head (id_1 k_1)...(id_n k_n)]$$

For each sugar special form, we give its syntax in its section header, provide an informal description of its usage and its rewritten form in terms of kernel constructs.

#### 2.4.1. (and $e_1...e_n$ )

An and expression performs a short-circuit "and" evaluation of  $e_i$  to  $v_i$ , returning #f if one of the  $v_i$  is #f, #t otherwise.

Rewrite Semantics

- #t (n = 0)
- (if  $e_1$  (and  $e_2...e_n$ ) #f)

2.4.2. (cond 
$$(e_1 e'_1)...(e_n e'_n)$$
 (clse  $e'_{n+1}$ ))

A cond expression is a multiple-way test expression. The tests  $e_i$  are successively evaluated to  $v_i$  and as soon as one (say j) returns #t (or else is reached), the value of  $e'_j$  is returned.

Rewrite Semantics

- $e'_{n+1}$  (n=0)
- (if  $e_1 e'_1$  (cond  $(e_2 e'_2)...(e_n e'_n)$  (else  $e'_{n+1}$ )))

2.4.3. (let\* 
$$((id_1 e_1)...(id_n e_n)) e$$
)

A let\* expression successively binds each  $id_i$  to the value  $v_i$  of  $e_i$  evaluated in an augmented environment that binds  $id_j$  to  $v_j$  for j in [1, i-1]. The value of e, evaluated in an augmented environment that binds  $id_i$  to  $v_i$ , is returned.

Rewrite Semantics

- $\bullet$   $\epsilon$  (n=0)
- (let ((id<sub>1</sub> e<sub>1</sub>)) (let\* ((id<sub>2</sub> e<sub>2</sub>)...(id<sub>n</sub> e<sub>n</sub>)) e))

2.4.4. (letrec 
$$((id_1 e_1)...(id_n e_n)) e$$
)

A letrec expression recursively binds each  $id_i$  to the value  $v_i$  of  $e_i$ . The value of e, evaluated in an augmented environment that binds  $id_i$  to  $v_i$ , is returned.

Rewrite Semantics

(let ((id (module (define  $id_1 e_1$ )...(define  $id_n e_n$ )))) (with id e))

where id is fresh.

#### 2.4.5. (match $e(pat_1 e_1)...(pat_n e_n)$ )

A match expression evaluates e to v and then performs a sequential match of v against the patterns  $pat_i$ . As soon as a match is found with a pattern  $pat_i$ , the value of  $e_i$ , evaluated in an environment in which the free variables of  $pat_i$  are bound to the appropriate components of v, is returned.

Rewrite Semantics

where id is fresh and the clause expansion function  $expand_{clause}(pat_1...pat_n, e_1...e_n, v, s, f)$  is defined by:

- (f v), if n = 0
- $expand_{exp}(pat_1, v, (s e_1), e')$  where e' is  $expand_{clause}(pat_2...pat_n, e_2...e_n, v, s, f)$ , otherwise.

The expression expansion function  $expand_{exp}(pat, v, s', f')$  is defined by:

- (if (= pat v) s' f'), if pat is a literal and = is the equality predicate defined on the type of the literal pat
- s', if pat is \_
- (let ((id v)) s'), if pat is id
- (ev (lambda  $(id_1...id_n)$  e') (lambda (x) f')), where the  $id_i$  and x are fresh and e' is  $expand_{pat}(pat_1...pat_n, id_1...id_n, s', f')$ , if pat is  $(e pat_1...pat_n)$ .

The pattern expansion function  $expand_{pat}(pat_1...pat_n, id_1...id_n, s', f')$  is defined by:

- s', if n=0
- $expand_{exp}(pat_1, id_1, e', f')$  where e' is  $expand_{pat}(pat_2...pat_n, id_2...id_n, s', f')$ , otherwise

2.4.6. (or 
$$e_1...e_n$$
)

An or expression performs a short-circuit "or" evaluation of  $e_i$  to  $v_i$ , returning #t if one of the  $v_i$  is #t, #f otherwise.

Rewrite Semantics

- #f (n=0)
- (if e<sub>1</sub> #t (or e<sub>2</sub>...e<sub>n</sub>))

**2.4.7.** 
$$id_1.id_2....id_n.id$$

An infix left-associative "dot" expression returns the value of id in the module that is the value of  $id_1.id_2...id_n$ .

Rewrite Semantics

- (with  $id_1 id$ ) (n = 1)
- (with id1 id2....idn.id)

#### 2.4.8. id1..id2

A "dotdot" expression denotes the description expression bound to  $id_2$  in the module  $id_1$ .

#### Rewrite Semantics

#### 2.4.9. $[e dx_1...dx_n]$

A  $\square$  expression returns the value of e projected on  $dz_1...dz_n$ .

#### Rewrite Semantics

$$(proj e dx_1...dx_n)$$

#### 2.4.10. (define head e)

A define expression with parenthesized or bracketed head respectively defines a function or a polymorphic value.

#### Rewrite Semantics

- (define head' (lambda  $((id_1 tx_1)...(id_n tx_n))$  e)), if head is (head'  $(id_1 tx_1)...(id_n tx_n)$ )
- (define head' (plambda  $((id_1 \ k_1)...(id_n \ k_n))$  e)), if head is  $[head' \ (id_1 \ k_1)...(id_n \ k_n)]$

#### 2.4.11.

(define-datatype [(id (id<sub>1</sub> 
$$k_1$$
)...(id<sub>n</sub>  $k_n$ )) | id]  
(id'<sub>1</sub>  $dx_{11}$  ... $dx_{1m_1}$ )...  
(id'<sub>n</sub>  $dx_{n1}$  ... $dx_{nm_n}$ ))

A define-datatype expression defines a possibly higherorder abstract type and a set of functions suited for creating and manipulating (via match) values of that type. A higher-order type definition introduces the following definitions in the current module binding (the case for a simple type is similar, with dlambda and plambda eliminated).

#### Rewrite Semantics

- (define-abstraction id(->>  $k_1...k_n$ )

  (dlambda  $((id_1 \ k_1)...(id_n \ k_n))$ (sumof  $(id'_1 \ (productof \ (L_1 \ dx_{11})...$ ( $L_{m_1} \ dx_{1m_1})))...$ ( $id'_p \ (productof \ (L_1 \ dx_{p1})...$ ( $L_{m_n} \ dx_{pm_n}))))))$
- (define-description id-rep (dlambda (( $id_1 \ k_1$ )...( $id_n \ k_n$ )) (sumof ( $id_1$  (productof ( $L_1 \ dx_{11}$ )... ( $L_{m_1} \ dx_{1m_1}$ )))... ( $id_p$  (productof ( $L_1 \ dx_{p1}$ )... ( $L_{m_p} \ dx_{pm_p}$ ))))))

```
• (define-typed id;
   (poly ((id_1 k_1)...(id_n k_n))
             (x_1 \text{ effect}) (x_2 \text{ effect}) (t \text{ type}))
    (-> fx..pure
          ((v (id id_1...id_n))
           (s (-> x_1)
                    ((id''_1 dx_{i1})...(id''_{m_i} dx_{im_i}))
           (f (-> x_2 ((v (id id_1...id_n))) t)))
          t))
    (plambda ((id_1 k_1)...(id_n k_n))
                 (x_1 \text{ effect}) (x_2 \text{ effect}) (t \text{ type}))
     (lambda
       ((v (id id_1...id_n))
        (s \leftarrow x_1 ((id''_1 dx_{i1})...(id'_{m_i} dx_{im_i})) t))
        (f(-> x_2)((v(id\ id_1...id_n))))))
      (tagcase (id-rep id_1...id_n)
                  (id-down\ v)
                  id'i
                  (lambda (v_t))
                     (s (extract (productof
                                        (L_1 dx_{i1})...
                                        (L_m, dx_{im})
                                      L_1)...
                        (extract (productof
                                        (L_1 dx_{i1})...
                                        (L_m, dx_{im_i})
                                     L_{m_i})))
                  (lambda(x)(fv))))
```

where  $L_i$ ,  $id'_i$ ,  $x_i$ , t, v,  $v_t$ , s, f and x are fresh.

#### 2.4.12. (do (id $e_0$ $e_i$ ) ( $e_i$ $e_r$ ) e)

A do expression is a loop expression. The expression e is iteratively evaluated, while the value of  $e_t$  is #1, in an environment in which id is initially bound to  $e_0$  and then to  $e_i$  in all subsequent iterations. Once  $e_t$  evaluates to #t, the value of  $e_\tau$  is returned.

Rewrite Semantics

where id is fresh.

2.4.13. (abs 
$$(id_1 \ id_2...id_n) \ k$$
)

An abs form with a list of identifiers denotes a sequence of abs forms for each  $id_i$ .

Rewrite Semantics

- (abs  $id_1 k$ ) (n=1)
- (abs  $id_1 k$ ) (abs  $(id_2...id_n) k$ )

2.4.14. (val 
$$(id_1 id_2...id_n) tx$$
)

A val form with a list of identifiers denotes a sequence of val forms for each  $id_i$ .

Rewrite Semantics

- (val  $id_1 tx$ ) (n=1)
- (val  $id_1 tx$ ) (val  $(id_2...id_n) tx$ )

#### 3. Standard Descriptions

The fx module defines the standard effects and standard types that are provided by every FX implementation. They fill out the framework introduced by the FX Kernel with a set of useful types and subroutines.

The FX standard effects are given first. The FX standard types and type constructors appear in order of increasing complexity. There is a section for each data type or type constructor, giving its kind, a brief overview of its purpose, the syntax of literals, a list of subroutines with their types, an informal semantics and description of error conditions. In the semantic description of a subroutine, arguments are denoted by the names appearing in the type of the subroutine.

#### 3.1. Pure effect

The pure effect is the effect of referentially transparent computations. It is already defined in the FX Kernel (cf. previous chapter).

#### 3.2. Init

effect

The init effect is the effect of computations that only initialize freshly allocated memory locations. It is already defined in the FX Kernel (cf. previous chapter).

#### 3.3. Read

effect

The read effect is the effect of computations that only read memory locations. It is already defined in the FX Kernel (cf. previous chapter).

#### 3.4. Write

effect

The write effect is the effect of computations that only write memory locations. It is already defined in the FX Kernel (cf. previous chapter).

#### 3.5. Unit

type

The unit type denotes the set of values of computations that only perform side-effects. It is already defined in the FX Kernel (cf. previous chapter).

There is one value of type unit: the literal #u.

#### 3.6. **Bool**

type

The bool type denotes the set of boolean values. It is already defined in the FX Kernel (cf. previous chapter).

There are two boolean literals: #t (for the true boolean) and #f (for the false boolean).

```
equiv? : (-> pure ((p bool) (q bool)) bool)
and? : (-> pure ((p bool) (q bool)) bool)
or? : (-> pure ((p bool) (q bool)) bool)
not? : (-> pure ((p bool)) bool)
```

Equiv? returns #t if p and q are both true or both false and #f otherwise. The subroutines and? and or? respectively return the logical "and" and logical "or" of p and q. Not? returns the negation of p.

#### 3.7. Int

type

The int type denotes the set of integers.

An integer literal is formed by an optional base prefix, an optional + or - sign (+ is assumed if omitted), and a non-empty succession of digits that are defined in the given base. There are four distinct base prefixes: #b (binary), #o (octal), #d (decimal) and #x (hexadecimal). If no prefix is supplied, #d is assumed.

```
(-> pure ((i int) (j int)) bool)
<
                  :
                            (-> pure ((i int) (j int)) bool)
>
                            (-> pure ((i int) (j int)) bool)
                  :
                            (-> pure ((i int) (j int)) bool)
<=
                 :
>=
                  :
                            (-> pure ((i int) (j int)) bool)
                 :
                            (-> pure ((i int) (j int)) int)
                            (-> pure ((i int) (j int)) int)
*
                  :
                  :
                            (-> pure ((i int) (j int)) int)
                  :
                            (-> pure ((i int) (j int)) int)
neg
                            (-> pure ((i int)) int)
                  :
remainder
                            (-> pure ((i int) (j int)) int)
                  :
modulo
                            (-> pure ((i int) (j int)) int)
absolute
                            (-> pure ((i int)) int)
```

The subroutines =, <, >, <= and >= respectively return #t if x is equal, less than, greater than, less than or equal to and greater than or equal to j and #f otherwise. The subroutines +, \* and - respectively return the sum, product and difference of i and j. / returns the truncated division of i by j. Weg returns the opposite of i. The subroutines remainder and modulo both return the rest of the number-theoretic integer division of i by j; they differ on negative arguments (the value returned by remainder has the same sign as i). Absolute returns the absolute value of i.

A dynamic error is signalled in case of division by zero or overflow. The range of integer values and subroutines is unspecified.

#### 3.8. Float type

The float type denotes the set of floating point numbers.

A float literal is formed by an optional + or - sign (+ is assumed if omitted), a non-empty succession of decimal digits, a decimal point, a non-empty succession of decimal digits and an optional exponent denoted by the letter E or e, an optional + or - sign (+ is assumed if omitted) and a sequence of decimal digits.

```
11=
                       (-> pure ((x float) (y float)) bool)
11<
               :
                       (-> pure ((x float) (y float)) bool)
11>
                :
                       (-> pure ((x float) (y float)) bool)
11<=
                       (-> pure ((x float) (y float)) bool)
                :
11>=
                       (-> pure ((x float) (y float)) bool)
11+
                       (-> pure ((x float) (y float)) float)
                :
11+
                :
                       (-> pure ((x float) (y float)) float)
11-
                       (-> pure ((x float) (y float)) float)
11/
                :
                       (-> pure ((x float) (y float)) float)
flneg
                :
                       (-> pure ((x float)) float)
flabs
               :
                       (-> pure ((x float)) float)
exp
                       (-> pure ((x float)) float)
log
                       (-> pure ((x float)) float)
                :
sqrt
                       (-> pure ((x float)) float)
sin
                :
                       (-> pure ((x float)) float)
COS
                :
                       (-> pure ((x float)) float)
tan
                :
                       (-> pure ((x float)) float)
asin
                :
                       (-> pure ((x float)) float)
acos
                       (-> pure ((x float)) float)
atan
                :
                       (-> pure ((x float)) float)
floor
                       (-> pure ((x float)) int)
ceiling
                :
                       (-> pure ((x float)) int)
truncate
                       (-> pure ((x float)) int)
round
                :
                       (-> pure ((x float)) int)
int->float
                       (-> pure ((x int)) float)
```

The subroutines f1=, f1<, f1>, f1<= and f1>= respectively return #t if x is equal to, less than, greater than, less than or equal to and greater than or equal to y and #f otherwise. The subroutines fl+, fl+, fl- and fl/ respectively return the sum, product, difference and division of x and y. Flneg returns the opposite of x. Flabs returns the absolute value of x. Exp returns e to the power of x. Log returns the natural logarithm (in base e) of x. Sqrt returns the square root of x. The subroutines sin, cos, tan, asin, acos and atan respectively return the sine, cosine, tangent, arcsine (within ]- $\pi/2,\pi/2$ ]), arccosine (within ]- $\pi/2,\pi/2$ ]) and arctangent (within  $]-\pi/2,\pi/2]$ ) of x. The subroutines floor and ceiling respectively return the largest and smallest integer not larger and smaller than x. Truncate returns the integer of largest absolute value not larger than (flabs x) and of same sign as x. Round returns the closest (even if tie) integer to x. Int->float returns the real x such that (floor x) = (ceiling x) = x.

A dynamic error is signalled in case of division by zero, overflow or underflow. The subroutines log and sqrt signal an error if x is not positive. The precision of floating point values and subroutines is unspecified: truncation may occur if the number of significant digits is too large.

#### 3.9. Char type

The char type denotes the set of characters.

A character literal is formed by a #\ prefix io:lowed by a character or an identifier followed by a delimiter. The list of allowed identifiers must include: backspace, newline, page, space and tab.

```
char=?
                         (-> pure ((c char) (d char)) bool)
char<?
                         (-> pure ((c char) (d char)) bool)
                    :
char>?
                         (-> pure ((c char) (d char)) bool)
char<=?
                    :
                         (-> pure ((c char) (d char)) bool)
char>=?
                         (-> pure ((c char) (d char)) bool)
char-ci=?
                         (-> pure ((c char) (d char)) bool)
                    :
                         (-> pure ((c char) (d char)) bool)
char-ci<?
char-ci>?
                         (-> pure ((c char) (d char)) bool)
                    :
char-ci<=?
                         (-> pure ((c char) (d char)) bool)
char-ci>=?
                         (-> pure ((c char) (d char)) bool)
char-alphabetic? :
                         (-> pure ((c char)) bool)
char-numeric?
                         (-> pure ((c char)) bool)
char-whitespace?
                         (-> pure ((c char)) bool)
char-lower-case?
                         (-> pure ((c char)) bool)
char-upper-case?
                         (-> pure ((c char)) bool)
char-upcase
                         (-> pure ((c char)) char)
char-downcase
                         (-> pure ((c char)) char)
char->int
                         (-> pure ((c char)) int)
int->char
                         (-> pure ((c int)) char)
```

The subroutines char=?, char<?, char>?, char<=? and char>=? respectively return #t if c is equal to, less than, greater than, less than or equal to and greater than or equal to d and #f otherwise; these tests are based on a total ordering of characters which is compatible with the ASCII standard on lower-case letters, upper-case letters and digits (without any interleaving between letters and digits). The subroutines char-ci=?, char-ci<?,

char-ci>?, char-ci<=? and char-ci>=? respectively return #t if c is equal to, less than, greater than, less than or equal to and greater than or equal to d and #f otherwise; these tests are case-insensitive. Char-alphabetic? returns #t when c is alphabetic; a character is alphabetic if its lower-case version is between #\a and #\z. Char-numeric? returns #t when c is a (decimal) digit. Char-whitespace? returns #t when c is a white space. The subroutine char-lower-case? (char-upper-case?) returns #t if c is between  $\#\a$  ( $\#\A$ ) and  $\#\z$  ( $\#\Z$ ). The subroutines char-upcase and char-downcase respectively return the upper-case and lower-case version of c; nonalphabetic characters remain unchanged. Char->int returns the index of c in the character ordering mentioned above. Int->char returns the character with ordering index c.

Int->char signals an error if c is not compatible with the character ordering.

#### 3.10. String

type

The string type denotes the set of mutable zero-based integer-indexed sequences of characters. Once created, a string is of constant length.

A string literal is formed by a double-quote ("), a sequence of characters (where \ is the escape character for itself and the double-quote character) and an ending double-quote.

make-string	:	(-> init ((length int) (c char))
string-length string-ref	:	string) (-> pure ((s string)) int) (-> read
string-set!	:	<pre>char) (-&gt; write      ((s string) (index int)           (new-c char))</pre>
string-fill!	:	unit) (-> write
string=?	:	<pre>(-&gt; read    ((s string) (t string))    bool)</pre>
string </td <td>:</td> <td>(-&gt; read ((s string) (t string)) bool)</td>	:	(-> read ((s string) (t string)) bool)
string>?	:	(-> read ((s string) (t string)) bool)
string<=?	:	(-> read ((s string) (t string)) bool)

string>=? (-> read ((s string) (t string)) bool) string-ci=? (-> read : ((s string) (t string)) bool) (-> read string-ci<? : ((s string) (t string)) bool) string-ci>? (-> read ((s string) (t string)) bool) string-ci<=? (-> read ((s string) (t string)) bool) string-ci>=? (-> read ((s string) (t string)) bool) substring (-> (maxeff init read) ((s string) (from int) (to int)) string) string-append : (-> (maxeff init read) ((head string) (tail string)) string) string-copy (-> (maxeff init read) ((s string))

Make-string allocates and returns a string of length characters c. String-length returns the length of s. String-ref returns the character of s that is at the index position. String-set! replaces in s the character at the index position with new-c and returns #u. String-fill! replaces each character of s with fill and returns #u. The subroutines string=?, string<?, string>?, string<=? and string>=? respectively return #t if s is lexicographically equal to, less than, greater than, less than or equal to and greater than or equal to t and #f otherwise. The subroutines string-ci=?, string-ci<?, string-ci>?, string-ci<=? and string>=? respectively return #t if s is lexicographically equal to, less than, greater than, less than or equal to and greater than or equal to t and #f otherwise; these tests are case-insensitive. Substring allocates and returns a string formed from the characters of s between the indices from and to (exclusive); if from and to are equal, then the substring returned is the empty string (""). String-append allocates and returns a string formed by the concatenation of head and tail. String-copy allocates and returns a string with the characters present in s.

string)

It is a dynamic error to try to access out-of-bounds elements of strings. Substring signals a dynamic error if from is not in [0,(string-length s)[, if to is not in [0,(string-length s)] and if from is not less than or equal to to.

#### 3.11. Sym

type

The sym type denotes the set of values that are solely defined by their name.

A symbol literal is formed by a left parenthesis ((), the keyword symbol, a case-insensitive identifier and a right parenthesis ()).

Sym->string allocates and returns a string corresponding to the name of s. String->sym returns the symbol with name s. Sym=? returns #t if s and t have the same name and #1 otherwise.

#### 3.12. Permutation

type

The permutation type denotes the set of one-to-one mappings on finite intervals of integers starting at 0. Other permutation operations are described with the vector operations (see below).

```
make-permutation :
                            (-> pure
                                 ((pi (-> pure
                                          ((from int))
                                          int))
                                  (length int))
                                permutation)
cshift
                             (-> pure
                                 ((length int) (offset int))
                                permutation)
identity
                             (-> pure
                      :
                                 ((length int))
                                permutation)
```

Make-permutation returns the permutation that maps every integer from in the interval [0,1ength[ to (pi from). Cshift returns the permutation that performs a circular shift (i.e. elements shifted out at one end are shifted in at the other end) on the interval [0,1ength[ by offset positions on the right if offset is positive and by (neg offset) positions on the left otherwise. Identity returns a permutation that maps every positive integer less than length to itself.

Make-permutation signals a dynamic error if length is not positive. It is a dynamic error if pi does not define a one-to-one mapping. The subroutines cshift and identity signal an error if length is not positive.

### 3.13. Refof (->> type)

The type (refof t) denotes the set of mutable references to values of type t. It is already defined in the FX Kernel (cf. previous chapter).

Ref allocates and returns a new reference with initial value valo. ^ returns the value stored in ref. := replaces the value stored in ref with val1 and returns #u.

### 3.14. Uniqueof

(->> type)

The type (uniqueof t) denotes the multiset of values of type t.

Unique allocates and returns a unique value from x; the init effect ensures that no memoization will be performed on calls to unique. Value returns the embedded value corresponding to u. Eq? returns #t when u1 and u2 have been created by the same call to unique.

#### 3.15. Listof

(->> type)

The type (listof t) denotes the set of mutable homogeneous lists of values of type t.

```
null
                      (poly ((t type))
                         (-> pure () (listof t)))
null?
                      (poly ((t type))
                         (-> pure ((list (listof t))) bool))
                      (poly ((t type))
cons
                         (-> init
                             ((car t) (cdr (listof t)))
                             (listof t)))
                      (poly ((t type))
car
                         (-> read ((list (listof t))) t))
cdr
                      (poly ((t type))
                         (-> read
                             ((list (listof t)))
                             (listof t)))
set-car! :
                 (poly ((t type))
                    (-> write
                        ((list (listof t)) (new t))
                        unit))
set-cdr! :
                 (poly ((t type))
                    (-> write
                        ((list (listof t)) (new (listof t)))
                        unit))
length
           :
                 (poly ((t type))
                    (-> read ((list (listof t))) int))
append
                 (poly ((t type))
                    (-> (maxeff read init)
                        ((front (listof t))
```

(rear (listof t)))

(listof t)))

list->string :

string->list :

```
TAVATSA
                     (poly ((t type))
                        (-> (maxeff init read)
                             ((list (listof t)))
                             (listof t)))
list-tail
                     (poly ((t type))
                        (-> read
                             ((list (listof t)) (minus int))
                             (listof t)))
list-ref
                     (poly ((t type))
                        (-> read
                             ((list (listof t)) (index int))
                             t))
map
                     (poly ((ti type) (t2 type) (e effect))
                        (-> (maxeff a init read)
                             ((f (-> a ((x t1)) t2))
                             (list (listof t1)))
                             (listof t2)))
for-each
                     (poly ((t1 type) (t2 type) (e effect))
                         (-> (maxeff e read)
                             ((f (-> \bullet ((x t1)) t2))
                              (list (listof t1)))
                             unit))
reduce
                     (poly ((t1 type) (t2 type) (e effect))
                         (-> (maxeff e read)
                             ((f (-> e ((x t1) (red t2)) t2))
                              (list (listof t1))
                              (seed t2))
```

t2))

string)

(-> (maxeff read init)

(listof char))

((chars (listof char)))

Mull returns the empty list. Mull? returns #t if the list is empty and #f otherwise. Cons allocates and returns a list with car as first element and cdr as remaining elements. Car returns the first element of list. Cdr returns the list after the first element of list. Set-car! replaces the first element of list with new and returns #u. Set-cdr! replaces the rest of list with new and returns #u. Length returns the number of elements in list. Append allocates and returns a list that is the concatenation of front and rear. Reverse allocates and returns a list with the elements of list in the reverse order. List-tail returns the sublist of list after omitting its first minus elements. List-ref returns the index-th element of list. Map allocates and returns a list that is obtained by consing the results of applying f on each element x of list from left to right. For-each applies f to each element x of list from left to right and returns #u. Reduce returns the result of the right-associative running (in red) applications of 1 with each element x of list, beginning with seed; seed is returned if list is empty. List->string allocates and returns a string made of chars. String->list allocates and returns a list made of chars.

It is a dynamic error to apply access operations such as car or cdr on the empty list. A dynamic error is signalled if set-car! or set-cdr! is applied to the empty list. A dynamic error is signalled if the index is out of range in list-ref or if minus is greater than the length of list in

```
(->> type)
3.16. Vectorof
The type (vector of t) denotes the set of mutable, zero-
based, integer-indexed, homogeneous vectors that contain
elements of type t. Once created, a vector is of constant
length.
make-vector
                            (poly ((t type))
                               (-> init
                                   ((length int) (value t))
                                   (vectorof t)))
vector-length
                            (poly ((t type))
                               (-> pure
                                    ((vector (vectorof t)))
                                   int))
vector-ref
                            (poly ((t type))
                               (-> read
                                   ((vector (vector of t))
                                    (index int))
                                   t))
vector-set!
                    :
                             (poly ((t type))
                                  (-> write
                                     ((vector (vector of t))
                                      (index int)
                                      (new t))
                                     unit))
vector-fill!
                             (poly ((t type))
                    :
                                 (-> write
                                    ((old (vectorof t))
                                     (new t))
                                    unit))
vector->list
                             (poly ((t type))
                                 (-> (maxeff init read)
                                    ((vector (vector of t)))
                                    (listof t)))
list->vector
                             (poly ((t type))
                                (-> (maxeff init read)
                                    ((list (listof t)))
                                    (vectorof t)))
                  (poly ((t1 type) (t2 type) (e effect))
vector-map :
                     (-> (maxeff e init read)
```

((f (-> e ((v t1)) t2))

(vectorof t2)))

(-> (maxeff e init read)

(vectorof u)))

vector-map2:

vector-reduce :

(vector (vectorof t1)))

(vector1 (vectorof t1))

(vector2 (vectorof t2)))

(-> (maxeff e read)

(seed u))

(poly ((t1 type) (t2 type) (u type) (e effect))

((f (-> e ((v1 t1) (v2 t2)) u))

(poly ((t type) (u type) (e effect))

(vector (vectorof t))

((f (-> e ((x t) (red u)) u))

list-tail.

```
scan
                          (poly ((t type) (e effect))
                             (-> (maxeff e init read)
                                 ((f (-> e ((x t) (y t)) t))
                                  (vector (vector of t)))
                                 (vectorof t)))
segmented-scan
                          (poly ((t type) (e effect))
                             (-> (maxeff e init read)
                                 ((f (-> o ((x t) (y t)) t))
                                  (segments (vectorof bool))
                                  (vector (vectorof t)))
                                 (vectorof t)))
permute
                          (poly ((t type))
                             (-> (maxeff init read)
                                 ((mapping permutation)
                                  (vector (vectorof t)))
                                 (vectorof t)))
compress
                        (poly ((t type))
                            (-> (maxeff init read)
                                ((selection (vector of bool))
                                  (vector (vectorof t)))
                                 (vectorof t)))
expand
                        (poly ((t type))
                             (-> (maxeff init read)
                                ((selection (vector of bool))
                                  (vector (vectorof t))
                                  (default (vectorof t)))
                                 (vectorof t)))
eoshift
                        (poly ((t type))
                            (-> (maxeff init read)
                                ((offset int)
                                 (vector (vectorof t))
                                 (default (vectorof t)))
                                (vectorof t)))
```

Make-vector allocates and returns a vector of length elements, each having the given value. Vector-length returns the number of elements in vector. Vector-ref returns the index-th element of vector. Vector-set! replaces the index-th value of vector with new and returns #u. Vector-fill! replaces each element of old with new and returns #u. Vector->list returns a list constructed from the elements of vector. List->vector allocates and returns a vector constructed from the elements of list. Vector-map allocates and returns a vector that is obtained by applying f to each element v of vector. Vector-map2 allocates and returns a vector that is obtained by applying f to each element v1 of vector1 and v2 of vector2. Vector-reduce returns the result of the right-associative running (in red) applications of f with each element of vector. Scan allocates and returns a vector in which the element of offset i-1 is the reduction by f of the first i elements of vector. Segmented-scan allocates and returns a vector that contains the reductions by f of subvectors of vector corresponding to each contiguous sequence of #f of segments. Permute allocates and returns a vector obtained by permuting vector according to mapping; specifically, if mapping maps x to y, then (vector-ref (permute mapping vector) y) is (vector-ref vector z). Compress allocates and returns a vector obtained by selecting from vector the elements that have a corresponding #t value in selection. Expand allocates and returns a vector obtained by replicating default, except for entries

in selection that are #t in which case the next available element of vector is chosen. Eoshift allocates and returns a vector obtained by performing an "End-Off" shift (i.e. element are shifted out at one end and default values are shifted in at the other end) of vector by offset positions on the right if offset is positive and by (neg offset) positions on the left otherwise.

The subroutines vector-ref and vector-set! signal a dynamic error if index is not in [0, (vector-length vector)]. It is a dynamic error for f not to be associative in vector-reduce, scan and segmented-scan. Segmented-scan signals a dynamic error if the lengths of segments and vector differ. Permute signals a dynamic error if the length of input differs from the domain of the mapping. Compress signals a dynamic error if the lengths of selection and vector differ. Expand signals a dynamic error if the length of selection and default differ.

#### 3.17. Sexp

type

The sexp type denote the set of values that are usually defined as "symbolic expressions". The type sexp is defined by:

Sexp=? (recursively) compares the two symbolic expressions s1 and s2 for equality; for each basic type, the appropriate equality function is used.

Values of type sexp can be introduced in programs by the "quote" symbol (') in front of a symbolic constant. A symbolic constant is either a literal, a sequence of symbolic constants between parentheses (preceded by a hash sign for vectors). The desugaring of a symbolic constant is defined by induction:

- if the symbolic constant is a literal l of type t (e.g., 1.3), then its desugaring is  $(t->sexp\ l)$  (e.g.,  $(float->sexp\ 1.3)$ ).
- if the symbolic constant is a sequence between parentheses, then the desugarings of the constituents are gathered in a list *l* of type (listof sexp) and its

desugaring is (list->sexp l). If the sequence is preceded by a hash sign (#), then a vector v of type (vectorof sexp) is gathered and its desugaring is (vector->sexp v).

#### 3.18. Stream

type

The type stream denotes the set of values that serve as sequenced source or sink of values of type char. For programming convenience, the fx module contains operations on streams supporting the sexp type.

standard-input : stream standard-output : stream

open-input-stream : (-> (maxeff init write)

((file string))
stream)

open-output-stream: (-> (maxeff init write)

((file string))

stream-write-sexp : (-> write

((output stream) (value sexp))

unit)

write-sexp : (-> write ((value sexp)) unit)

stream-write-char : (-> write

((output stream) (value char))

unit)

write-char : (-> write ((value char)) unit)
stream-read-sexp : (-> write ((input stream)) sexp)

read-sexp : (-> write () sexp)

stream-read-char : (-> write ((input stream)) char)

read-char : (-> write () char)

stream-char-eof? : (-> write ((input stream)) bool)
stream-sexp-eof? : (-> write ((input stream)) bool)

close-stream : (-> write ((st stream)) unit)

error : (poly ((t type))
(-> write

((message string))

t))

Standard-input and : andard-output are implementation-defined streams (usually connected to the user terminal) on which input and output operations can be performed, respectively. Open-input-stream allocates and returns an input stream connected to the file. The interpretation of the string file is implementation-dependent. Stream-read-sexp and stream-read-char return the first value of the input stream. Read-sexp and read-char return the first value of the standard-input-stream. Open-output-stream allocates and returns an output stream connected to the file. Again, the interpretation of the string file is implementation-dependent. Stream-write-sexp and stream-write-char send the value to the output stream and return #u. Write-sexp and write-char send the value to the standard-output stream. Read operations have a write effect because they change the state of the stream. Stream-char-eof? returns #t if no more characters can be read from the input, #f otherwise. Stream-sexp-eof? returns #t if the end of the input will be reached before the start of the next s-expression, #f otherwise. Thus stream-sexp-eof? returns #1 if there is only an incomplete s-expression

at the end of the stream. Close-stream closes the stream st and returns #u. Both stream-sexp-eof? and stream-char-eof? return #t when applied to closed streams. Error prints its message on standard-output and signals a dynamic error.

Open-input-stream and open-output-stream signal a dynamic error if the file cannot be opened. It is a dynamic error to perform any operation (apart from testing for end of file) on a closed stream. It is a dynamic error to perform a read operation on an input stream if (stream-char-eof? input) is true. It is a dynamic error to perform a stream-sexp-read operation on an input stream if (stream-sexp-eof? input) is true. A dynamic error is signalled on attempts to read from a stream opened for output and on attempts to write to a stream opened for input. It is a dynamic error to apply an -eof? predicate to an output file. A dynamic error is signalled if a malformed s-expression is encountered by read-sexp or stream-read-sexp.

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# ALPHABETIC INDEX OF DEFINITIONS OF CONCEPTS, KEYWORDS, AND PROCEDURES

^, 9, 19	char-upper-case?, 17
• •	char-whitespace?, 17
*, 17	char=?, 17
<, 17	
>, 17	character, 3
	extended, 3
>=, 17	white space, 3
<=, 17	close-stream, 22
	comment, 4
+, 17	compress, 21
	cond, 14
<b>-</b> , 17	
	cons, 19
/, 17	conventions, 4
	cos, 17
:=, 9, 19	cshift, 19
=, 17	defabs, 11
	defdatatype, 15
abs, 7, 16	defdesc, 11
absolute, 17	•
acos, 17	define, 11, 15
and, 14	deftyped, 11
and?, 16	delimiter, 3
append, 19	desc, 7
application	dfunc, 5
	digit, 3
description, 6	dlambda, 6
value, 9	do, 15
asin, 17	dot
atan, 17	
	notation, 14
begin, 9	dot-dot
bool, 6, 16	notation, 15
bool->sexp, 21	
• /	effect, 5
car, 19	environment, kind, 5
cdr, 19	environment, type, 5
ceiling, 17	eoshift, 21
char, 17	eq?, 19
char>, 17	- ·
•	equiv?, 16
char>?, 17	error, 22
char>=?, 17	errors
char<=?, 17	dynamic, 4
char->int, 17	static, 4
char->sexp, 21	exp, 17
char-alphabetic?, 17	expand, 21
char-ci>, 17	expansive, 10
char-ci>?, 17	expression
char-ci>=?, 17	·
char-ci<=?, 17	description, 5
	kind, 5
char-ci=?, 17	value, 5
char-downcase, 17	extend, 9
char-lower-case?, 17	
	extract, 10
char-numeric?, 17	extract, 10
char-numeric?, 17 char-upcase, 17	f1*, 17

41 4 17	
f1<, 17	neg, 17
11>, 17	not?, 16
<b>11&gt;=</b> , 17	notations, 4
<b>fl&lt;=</b> , 17	not_expansive, 10
<b>11+</b> , 17	null, 19
<b>11-</b> , 17	null?, 19
<b>11/</b> , 17	number, 3
<b>11=</b> , 17	•
flabs, 17	open, 12
flneg, 17	open-input-stream, 22
float, 17	open-output-stream, 22
float->sexp, 21	or, 14
floor, 17	or?, 16
for-each, 20	<b>,</b>
FV, 5	permutation, 19
, .	permute 21
identifier, 4	plambda, 12
description, 6	poly, 7
reserved, 4	product, 12
value, 9	productof, 7
identity, 19	proj, 12
if, 10	projection
inferable, 6	implicit, 12
init, 6, 16	$\Box$ , 15
int, 16	
int->char, 17	pure, 6, 16
int->float, 17	read, 6, 16
int->sexp, 21	read-char, 22
200 - 20ap, 21	read-sexp, 22
lambde, 10	reduce, 20
le: n, 19	
let, 10	ref, 9, 19
let*, 14	refof, 6, 19
letrec, 14	remainder, 17
letter, 3	Reserved identifiers, 4
list->sexp	reverse, 20
list->string, 20	round, 17
list->vector, 20	scan, 21
list-ref, 20	· · · · · · · · · · · · · · · · · · ·
list-tail, 20	segmented-scan, 21
listof, 19	select, 8
literal, 4, 8	set-car!, 19
load, 11	set-cdr!, 19
•	sexp, 21
log, 17	sexp=?, 21
make-permutation, 19	sin, 17
make-string, 18	sqrt, 17
make-vector, 20	standard-input, 22
map, 20	standard-output, 22
match, 14	store, 8
maten, 14 maxeff, 7	stream, 22
	stream-char-eo1?, 22
memoization, 2	stream-read-char, 22
module, 11	stream-read-sexp, 22
moduleof, 7	stream-sexp-eof?, 22
modulo, 17	stream-write-char, 22
	stream-write-sexp, 22
	<b>4</b> / -

string, 18

string>, 18 string>?, 18 string>=?, 18 string<=?, 18 string->list, 20 string->sexp, 21 string->sym, 19 string-append, 18 string-ci>, 18 string-ci>?, 18 string-ci>=?, 18 string-ci<=?, 18 string-ci=?, 18 string-copy, 18 string-fill!, 18 string-length, 18 string-ref, 18 string-set!, 18 string=?, 18 subr, 6 substitution, 4 substring, 18 sum, 12 sumof, 8 sym, 18 sym->sexp, 21 sym->string, 19 sym=?, 19 tagcase, 13 tan, 17 the, 13 token, 3 truncate, 17 type, 5 unique, 19 uniqueof, 19 unit, 6, 16 unit->sexp, 21 val, 7, 16, 19 value, 8 variable bound, 4 free, 4, 5 vector->list, 20 vector->sexp, 21 vector-fill!, 20 vector-length, 20 vector-map, 20 vector-map2, 20 vector-reduce, 20 vector-ref, 20 vector-set!, 20

vectorof, 20

with, 13 write, 6, 16 write-char, 22 write-sexp, 22